Hypothesis assignment and partial likelihood averaging for cooperative estimation

Presented by: Parth Paritosh
Supervised by: Nikolay Atanasov, Sonia Martínez

University of California San Diego (UCSD)
Mechanical and Aerospace Engineering

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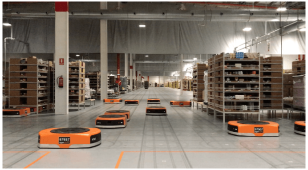
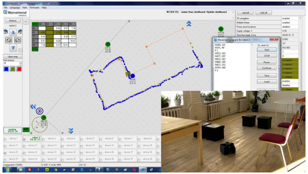
The ubiquity of distributed sensing infrastructure





Sensor network operations

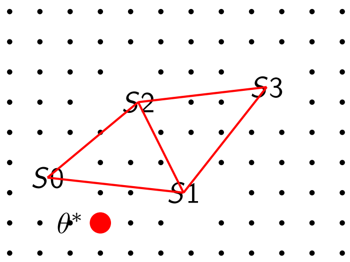
Physical	Sensor selection, communication infrastructure
Data	Storage and retrieval, Clustering
Inference	Estimation, Optimization and Control

Requirements

- Reliability, Security and Robustness
- Computational, storage and energy efficiency
- Temporal variations and Network size

Problem setup

- Discrete hypothesis space: Θ
- True hypothesis location(s): θ^*
- Sensing agents $(S0, \cdots, S3)$
- Local communication, (Weighted adjacency matrix: A)



- How to find the probability of true source location with decentralized communication network?

Existing research

- Jadbabaie et al.[1](2012): Opinion pooling with weighted sum updates in communication graphs
- Nedich et al.[2](2017): Convergence rates of geometric averaging of inferences for the decentralized communication problem
- Atanasov et al.[3](2014): Gaussian filtering algorithm based on geometric averaging

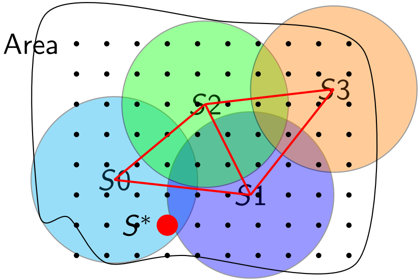
[3] Nedić, Angelia, Alex Olshevsky, and César A. Uribe. "Fast convergence rates for distributed non-bayesian learning." IEEE Transactions on Automatic Control 62.11 (2017): 5538-5553.

[3] Jadbabaie, Ali, et al. "Non-Bayesian social learning." Games and Economic Behavior 76.1 (2012): 210-225.

[3] Atanasov, Nikolay, et al. "Joint estimation and localization in sensor networks." 53rd IEEE Conference on Decision and Control. IEEE, 2014.

Problem setup

- Discrete hypothesis space: Θ
- True hypothesis location(s): θ^*
- Sensing agents $(S0, \cdots, S3)$
- Local communication
- Restricted observation space
- Local storage: Θ_i



Sensors $S0, S1, S2, S3$ learning the source S^* location as a probability over the Area.

The research questions

- Estimating source location probability in distributed communication and storage settings:

RQ1 Distributed storage: How to assign the discrete space (Θ_i) tracked by each agent?

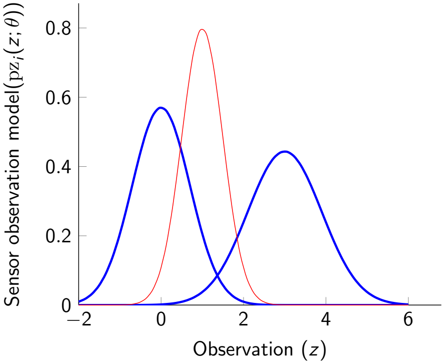
RQ2 Local communication: How to find true source location probability estimates $(p_i(\theta), \forall \theta \in \Theta_i)$ for each agent?

RQ1: Agent space assignment

Assignment objective: Intuition

Maximize the diversity in sensor observation models($\mathrm{pz}_i(z;\theta)$) at each hypothesis.

How to choose the sensors observing any hypothesis?



Assignment objective: Formulation

Diversity maximization

- Choosing sensor observation models over larger observation domain.
- Pairs of observation models with high divergence and entropy term.

$$F(\mathcal{G}_{\boldsymbol{\theta}}) = \sum_{\substack{i,j \in \mathcal{V}(\boldsymbol{\theta}) \\ (i,j) \in \mathcal{E}}} \mathrm{D_{KL}}(\mathrm{pz}_i(\cdot|\boldsymbol{\theta}), \mathrm{pz}_j(\cdot|\boldsymbol{\theta})) + \sum_{i \in \mathcal{V}(\boldsymbol{\theta})} \mathrm{H}(\mathrm{pz}_i(\cdot|\boldsymbol{\theta})). \tag{1}$$

Objective function defined over sets of subgraphs $\mathcal{G}_{\boldsymbol{\theta}}, \forall \boldsymbol{\theta} \in \Theta$ as

$$\max_{\{\mathcal{G}_{\boldsymbol{\theta}}\}} \sum_{\boldsymbol{\theta} \in \Theta} F(\mathcal{G}_{\boldsymbol{\theta}})$$

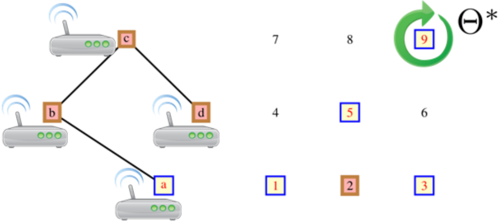
Sub-network assignment constraints

Spatial Coverage

Every hypothesis is observed by one of the sensors.

Limited observation space

Limit the observation space for each agent.



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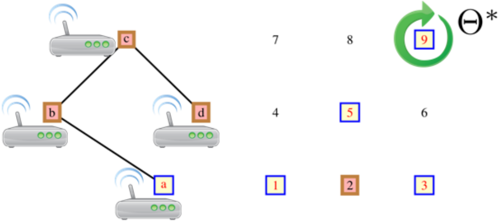
Subgraph connectivity

- Assign connected graphs for learning source probability at each hypothesis.
- NP-hard constraint.



Sub-network assignment as integer optimization

$y_{i,v}, b_{ij,v}$: Inclusion of sensor i and edge (i,j) in sub-network observing θ_v
$f^i_{ij,v}$: Flow variable for sensor i on edge (i,j) in sub-network observing θ_v

$$\sum_{v=1}^{m}\left[\max_{\mathbf{y}_v,\mathbf{b}_v}\sum_{(i,j)\in\mathcal{E}} b_{ij,v}\,\mathrm{D_{KL}}(\mathrm{pz}_i(z|\boldsymbol{\theta}_v),\mathrm{pz}_j(z|\boldsymbol{\theta}_v)) + \sum_{i=1}^{n} y_{iv}\,\mathrm{H}(\mathrm{pz}_i(z|\boldsymbol{\theta}_v))\right]$$

$$\sum_{v=1}^{m} y_{iv} \le m_i, \quad \forall i \in \{1,\ldots,n\}, \qquad \text{(Cardinality)}$$

$$\sum_{i} y_{iv} \ge 1, \quad \forall v \in \{1,\ldots,m\}. \qquad \text{(Coverage)}$$

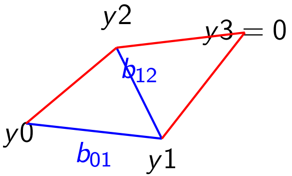
Tackling connectivity constraint

$$\sum_{ij \in \mathcal{E}} b_{ij,v} = \sum_{i=1}^{n} y_{iv} - 1, \qquad \text{(Tree selection)}$$

$$b_{ij,v} \leq y_{iv}, y_{jv}, \forall ij \in \mathcal{E}, \qquad \text{(Edge-node coupling)}$$

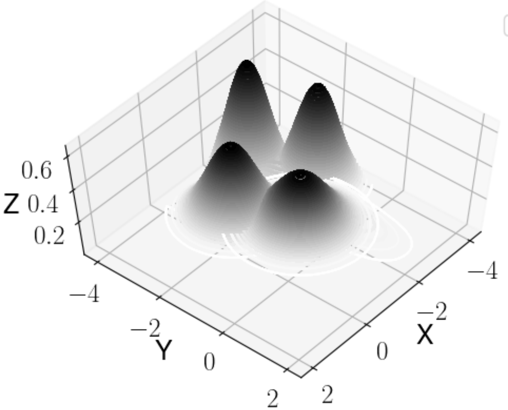
$$f^{i}_{ij,v} + f^{j}_{ij,v} = 2, \qquad \text{(Edge flow)}$$

$$\sum_{j \in \mathcal{N}_i} f^{j}_{ij,v} \leq 2 - \frac{2}{\sum_{i=1}^{n} y_{iv}}. \qquad \text{(Maximum average degree)}$$



Maximum average degree: $\left(\max_{H} \frac{\#edges}{\#nodes}\right)$, where H is subgraph of G. $2(k-1)/k$ for a connected tree of k nodes.

Illustration



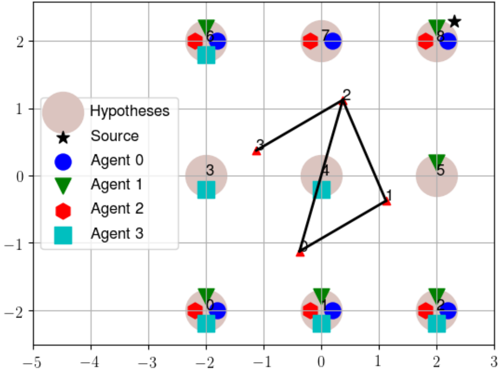


Figure: (a) True distance based Gaussian observation models $pz_i(\cdot|\boldsymbol{\theta}^*)$ for 4 agents. (b) Sensor-hypothesis assignment with number of hypotheses at each sensor limited to 6.

RQ2: Partial likelihood averaging algorithm

Algorithm: Partial likelihood averaging

Sensor i receiving observations $\{\boldsymbol{z}_{i,t}\}_{t=1}^{T}$ to combine inferences with distributed communication matrix $(\boldsymbol{A}(\boldsymbol{\theta}))$

Output: posterior probability $p_{i,T}$ over Θ_i

$\mu_{i,0}(\boldsymbol{\theta}) \leftarrow p_{i,0}(\boldsymbol{\theta}), \forall \boldsymbol{\theta} \in \Theta_i$ *% Initialization*

for $t \in \{1, \ldots, T-1\}$ **do**

  for $\boldsymbol{\theta} \in \Theta_i$ **do**

    % Geometric update

    $\mu_{i,t+1}(\boldsymbol{\theta}) = \prod_{j \in \mathcal{N}_i} \mu_{j,t}(\boldsymbol{\theta})^{A(\boldsymbol{\theta})_{ij}} \, \mathrm{pz}_i(\boldsymbol{z}_{i,t} | \boldsymbol{\theta})$

% Normalization factor

$Z_{i,T+1} = \sum_{\boldsymbol{\theta} \in \Theta_i} \mu_{i,T+1}(\boldsymbol{\theta}) + \sum_{\boldsymbol{\theta} \in \Theta \setminus \Theta_i} \mu_{j,T+1}(\boldsymbol{\theta})$

$p_{i,T}(\boldsymbol{\theta}) = \mu_{i,T}(\boldsymbol{\theta}) / Z_{i,t+1} \, \forall \boldsymbol{\theta} \in \Theta_i$

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Assumptions

Sensor data reception

Assume that if the true data generating pdf for agent i, $f_i^*(\boldsymbol{z}) > 0$ for some $z \in \mathbb{R}^{d_z}$, then $1 \geq \bar{\alpha} \geq \mathrm{pz}_i(z|\boldsymbol{\theta}) \geq \underline{\alpha} > 0$, for all $\boldsymbol{\theta} \in \Theta_i$ and some constants $\bar{\alpha}$, $\underline{\alpha}$. Note that $\bar{\alpha}$ exists for any pdf.

Other assumptions

Static graph The undirected graph $\mathcal{G}$ describing the agent communication is static and time-invariant.

Coverage Each hypothesis $\boldsymbol{\theta} \in \Theta$ is observed by at least one agent, i.e., $|\mathcal{V}(\boldsymbol{\theta})| \geq 1$.

Initial probabilities Every agent i has an initial likelihood $p_{i,0}(\boldsymbol{\theta}) > 0$, $\forall \boldsymbol{\theta} \in \Theta_i$.

Proof sketch

1. Consensus: All agents probability estimates converge to same value at all hypotheses.

$$\lim_{t\to\infty} A(\theta)^t = \frac{1}{n}\mathbb{1}\mathbb{1}^\top \qquad \text{(Doubly stochastic matrix)}$$

$$\lim_{t\to\infty} p_{i,t}(\theta) = p^*(\theta) \qquad \text{(For agents observing } \theta\text{)}$$

2. Convergence: The probability of an incorrect hypothesis is almost surely zero.

$$\lim_{t\to\infty} \frac{p_{i,t}(\theta_1)}{p_{i,t}(\theta_2)} \to 0 \qquad \text{(When only } \theta_2 \text{ is optimal)}$$

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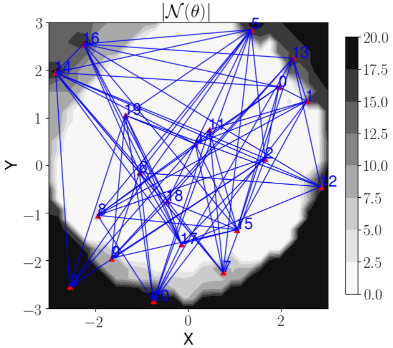
Assignment in 20-node network

For n agents, E edges and m hypotheses

- #variables : $\mathcal{O}(m(n+3|E|))$
- #constraints : $n+2m+4m|E|$

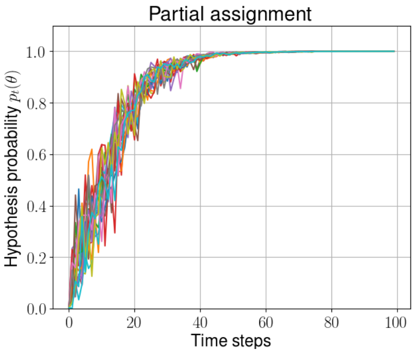


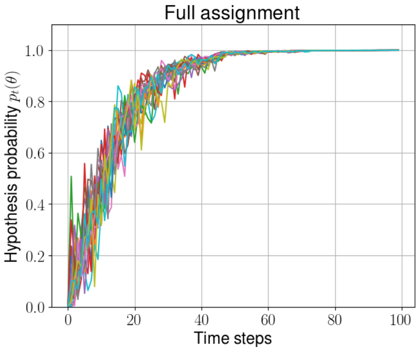
Number of agents
assigned at each hypothesis

* 20 agent connected graph in space with 300 discrete hypotheses
* Distance based observation model assigns more agents to low coverage areas

Convergence in 20-node network

* Comparing estimated probability at source with almost 1/3 communication and memory at each sensor:





Summary

- Matching each hypothesis to connected communication sub-networks based on diversity in observation models
- Geometric update based algorithm for finding inference
- Consensus among agents estimates at all hypotheses
- Proof based on comparing estimates at true and false hypotheses

Thank you